

# Measuring the World

Math for Elementary and Middle School  
Teachers

by  
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# Appendix 1: Tables with a spreadsheet program: Microsoft Excel and Open Office Basics

The commands described in this section apply for both Microsoft Excel and the free, open source software OpenOffice Spreadsheet. Many also work in the GeoGebra spreadsheet window, in Google Docs, and in the Apple program Pages. Screen snapshots are from the Macintosh version of Excel 2007. Other versions may have commands or buttons in different places; experiment with your program to find the right procedure (or read the online help.)

## A. What do spreadsheets do?

A *spreadsheet program* is a computer program that allows you to enter, arrange, and compute with data in tables. At first glance, it may seem that spreadsheets are merely a good way to keep organized (for example, in keeping records of your students' grades). In this section, you will begin to explore the possibilities for spreadsheets in algebraic thinking.

This appendix will explain how to do things in the following categories:

- Entering data
- Formulas
- Formatting (making it readable and pretty)
- Graphs (called Charts in spreadsheet programs)

The most important aspects for this course are Formulas and Graphs. This will not be a comprehensive course in Excel; it's just some of the features that are important for teaching and learning math.

Here's a spreadsheet table. At first glance, it looks like a neatly typed set of data about cheese weights and prices.

	A	B	C	D
1	Cheese (lb.)	Cost (\$)		
2	1	\$ 4.95		
3	2	\$ 9.90		
4	3	\$ 14.85		
5	7	\$ 34.65		
6	9.2	\$ 45.54		
7				
8				

Figure 1. Costs of various amounts of cheese, at \$4.95 per pound.

However, if you look at what's actually in the cell\*, you see formulas:

	A	B
1	Cheese (lb.)	Cost (\$)
2	1	=A2*4.95
3	2	=A3*4.95
4	3	=A4*4.95
5	7	=A5*4.95
6	9.2	=A6*4.95
7		

Figure 2. The formulas in the cheese table.

The cost of each weight of cheese is computed by the program: the weight of cheese is in column A, and it is multiplied by \$4.95/lb. to get its cost. Moreover, if you change the weights of cheese, the spreadsheet dynamically recomputes the costs, without your having to do anything.

In setting up this table, only the first formula was typed; you'll find out how to get the others shortly. As you learn techniques, keep in mind this general principle:

### Get the program to do as much work for you as possible!

In a small table, sometimes this takes more thinking than just doing the work by hand. But the point of this course is to think in increasingly sophisticated ways, not to do lots of typing or hand computation.

**Convention for these instructions:** a list of commands like "Format -> Column -> Autofit selection" means pull down the Format menu, choose Column, then choose Autofit selection.

\* Type Control-backquote (Press control at the same time as the tilde key without Shift; the key is usually to the left of 1.)

## B. Entering data

### Addresses of cells: rows and columns

A spreadsheet file is in the form of a grid style table. The rows are numbered, and the columns are lettered. In the cheese table above, the heading “Cheese” is in cell A1, and the heading “Cost” is in B1. The rest of column A (cells A2 through A6) list weights of cheese, and the rest of column B list the corresponding prices.

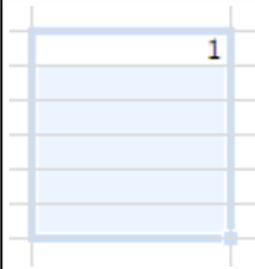
To type something in a cell, click on the cell and type. To move to a different cell, click in the other cell. Alternatively, Return/Enter to goes down one row; Tab goes right one cell; arrow keys go in the direction of the arrow.

To change something in a cell, click on it once and type in the space at the very top of the screen; alternatively, double click and type in the cell.

On the Edit menu, you can: copy or cut and paste, and add or delete cells, rows, or columns.

### Shortcuts for entering data in a pattern

If you have data without a particular pattern, you’ll just have to type it in. But if you wanted to enter, say, 1, 2, 3, 4, ... down the Cheese column, use a Fill command:

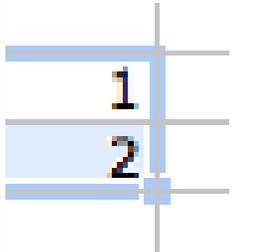
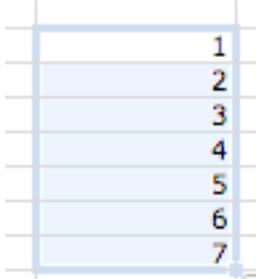
Type the first number in, then return.	
Select the cells you want the pattern to fill.	

<p>Edit -&gt; Fill -&gt; Series</p>	
<p>Then make the selections you want.</p>	

**Exercises.** Find out what the options do by trying various things: change the step value, try Growth instead of Linear. What happens if you choose AutoFill?

Try Edit -> Fill -> Down instead of Series. What happens?

**A further shortcut: using the fill handle.**

<p>When you select a cell or range of cells, there is a tiny square in the lower right corner called the fill handle.</p>	
<p>When you put the cursor over the fill handle, it changes to a + shape. Use that to drag out the region you want to fill. The cells are filled according to the pattern of the original cells you selected.</p>	

**Exercises.** What happens if you only type a number in one cell, then use the fill handle? What patterns can Excel figure out? Try some others: the doubling series 1, 2, 4 (see if it will figure out 8, 16, 32, etc.); try starting with 5, 8; try 1, 2, 1, 2 (will it continue alternating 1 and 2? What happens if you drag to the right, instead of down? Can you figure out how to drag left or up?

The moral of this exercise is that you are better at figuring out patterns than Excel is. When you use a shortcut to fill by a pattern, make sure to check that it did what you meant. Some other methods for generating patterns will be described in the Formulas section, below.

### Made a mistake?

Use Edit -> Undo or its keyboard shortcut, Control-Z.

If you need to wipe out a whole region of cells, select them, then use Edit -> Clear (or Control-B).

If you need some more rows between ones you have already filled, put the cursor in the row below the place you want to insert rows, then do Insert -> Rows. If you want several rows, highlight that many cells/rows below where you want the new rows, then Insert -> Rows.

There is extensive online help: just pull down the Help menu. If you can't figure something out, you can sometimes find help by doing a web search.

## C. Formulas.

This is one of the places Excel can really save you work.

A formula is a set of instructions that tell Excel how to compute what to put in the cell, rather than typing the number in explicitly.

- **To enter a formula**, you must start with an = sign.

- **Arithmetic operations:** Plus and minus are the usual keys on the keyboard. For multiplication, use \* (shift the 8 key). For division, use the slash (to the right of the period on the keyboard), like a fraction bar. Use ^ (shift 6) for exponents:  $7^2$  is entered as  $7^2$ . Excel knows order of operations.

**Example 1.** Type “=3\*5-2\*4” in a cell (don’t type the quotation marks) and hit return. What number appears, and why?

- **Operations involving the contents of other cells:** To multiply by whatever is in a specified cell, such as B2, just type the address (the technical terms is “cell reference”), rather than the number that’s in the cell. Then if you decide to change the number in B2, you won’t have to change your formula.

**Example 2.** The spreadsheet below keeps track of the money made by selling tickets at Saturday concerts. Kids’ tickets cost \$5, and adults’ cost \$8.

	A	B	C	D	E	F
1		Ticket sales: kids \$5, adults \$8				
2	Date	# of kids	# of adults	Money from kids	Money from adults	Total
3	8/14	25	26	=B3*5		
4	8/21	16	38			
5	8/28	7	17			
6	9/4	35	5			

To get Excel to find the money made from kids’ tickets on 8/14 (footnote<sup>†</sup>), tell it to multiply whatever is in column B by 5 by typing “=B3\*5”. Here’s what it shows after you hit return:

	A	B	C	D	E	F
1		Ticket sales: kids \$5, adults \$8				
2	Date	# of kids	# of adults	Money from kids	Money from adults	Total
3	8/14	25	26	\$125.00		
4	8/21	16	38			
5	8/28	7	17			
6	9/4	35	5			

**Shortcut:** while typing a formula, click in cell B3 instead of typing B3.

**Warning about the shortcut:** If you are editing/typing in a cell, and you try to get out of it by clicking another cell, it will type the address of that cell in the one you were trying to leave! Instead, use Return, Tab, or an arrow key.

After the formula is entered, if you click once in D3, the formula appears in the text field at the top of the screen. If you double-click, the formula appears in the cell. You can change it if you like.

- **Copying the same formula, but with different inputs:** You don’t have to retype this formula in D4, D5, and D6. Copy D3, then paste it into D4. Check that the amount is correct for date 8/21. Double click and carefully inspect the formula: it says  $B4*5$ .

<sup>†</sup> Why did the program leave the date as 8/14, instead of dividing 8 by 14?

**Key feature of spreadsheets:**  
**When you copy a formula, all the addresses change.**

For example, if you copy a formula down 2 cells and right 1, all the row numbers in any cell references will increase by 2, and all the column letters will increase by 1.

Using the fill handle will also work to copy the formula vertically or horizontally.

	A	B	C	D	E	F	
1		Ticket sales: kids \$5, adults \$8					
2	Date	# of kids	# of adults	Money from kids	Money from adults	Total	
3	8/14	25	26	\$125.00			
4	8/21	16	38	\$80.00			
5	8/28	7	17	\$35.00			
6	9/4	35	5	=B6*5			

**Exercise:** Figure out how to easily calculate the amount of money from adult tickets, and total for each date.

**Formulas for generating patterns.** If filling doesn't work for a pattern you can see, but Excel can't, use a formula. For example, to get the sequence of Fibonacci numbers 1, 1, 2, 3, 5, 8, 13, ..., type in the first two values 1 and 1, and in the next cell, type the sum of the addresses:

H	I	J	K	L	M	N
1	1	=H1+I1				

Now fill across, and the formula will be copied, with addresses changed appropriately.

H	I	J	K	L	M	N	O	P	Q
1	1	2	3	5	8	13	=M1+N1		

## Functions: built-in formulas

Excel has lots of built-in functions. Some useful ones:

Name	Usage	Result
square root	=sqrt(64)	8
sum	=sum(H1:N1) (in example above)	1+1+2+3+5+8+13=33
average	=average(H1:K1)	(1+1+2+3)/4 = 7/4 = 1.75
	=average(45, 57, 32)	44.67
pi	=pi()	(Don't put anything in the parentheses.) 3.14159

Some functions are logical: they answer true or false. These include equal to (=), greater than (>), less than (<), and not equal to (<>).

**Example 3.** The total you paid for one item, including 8% sales tax, was \$17.37. How much was the item without tax? Use the formulas  $=C2=17.37$  in cell D2 and  $=C2>17.37$  in cell E2 get Excel to check automatically if the result of a guess and check calculation is equal to 17.37, and whether it's bigger than 17.37.

	A	B	C	D	E
1	price	tax @ 8%	total	Total = \$17.37?	Total > \$17.37?
2	16	=A2*0.08	=A2+B2	=C2=17.37	=C2>17.37

Here are the results for three test values for the price

	A	B	C	D	E
1	price	tax @ 8%	total	Total = \$17.37?	Total > \$17.37?
2	\$16.00	\$1.28	\$17.28	FALSE	FALSE
3	\$17.00	\$1.36	\$18.36	FALSE	TRUE
4	\$16.50	\$1.32	\$17.82	FALSE	TRUE

### Absolute vs. relative cell references

When you copy a formula, all the cell addresses are changed to reflect the copying. For instance, if you copy a formula into a cell 2 right and 3 down from its original location, all the addresses are changed 2 right and 3 down. Sometimes this isn't what you want. For example, maybe there's one cell (say, C2) at the top where you type today's price for the catch of the day at your restaurant, and you use this number in a formula copied several places in your spreadsheet. To stop Excel from changing all or part of the address:

- put a \$ sign before either the row number or column letter, or both, in a cell address to prevent that part of the address from changing when copied.

So if the price for the catch of the day is in C2, type  $\$C\$2$  to refer to the exact location C2. The addresses  $\$C$  and  $\$2$  are called *absolute* cell references; they never change. Just plain addresses that change when copied, like C and 2, are called *relative* cell references. The reference  $\$C2$  will always be in column C, but the row number will change when copied. Similarly,  $C\$2$  will always be in row 2, but the column letter will change when copied.

### Grid tables in Excel with absolute cell references.

In a grid table, the input variables are usually in the first (top) row and first (left) column. To make an addition table, the formula should say: add the number in the same row, left column, and the number in the same column, top row (the headings). Here is the appropriate formula and result:

	A	B
1	+	0
2	0	=A2+B1

	A	B
1	+	0
2	0	0

But if you copy or fill the formula across row 2, look what happens: the addresses are changed so that they say: add the number just to the left with the number in row A. The result is not what we intended.

	A	B	C	D	E	F
1	+	0	1	2	3	4
2	0	=A2+B1	=B2+C1	=C2+D1	=D2+E1	=E2+F1

	A	B	C	D	E	F
1	+	0	1	2	3	4
2	0	0	1	3	6	10

To fix this, use the \$ notation. We want to always add the number in Column A with the number in Row 1. The formula to compute 0+0 in cell B2, should be:

$$= \$A2 + B\$1$$

Now copy/fill the formula to the right, then down. Notice how the columns in the first term of the formula stay A, and rows in the second term stay 1.

	A	B	C	D	E	F
1	+	0	1	2	3	4
2	0	= \$A2 + B\$1	= \$A2 + C\$1	= \$A2 + D\$1	= \$A2 + E\$1	= \$A2 + F\$1
3	1	= \$A3 + B\$1	= \$A3 + C\$1	= \$A3 + D\$1	= \$A3 + E\$1	= \$A3 + F\$1
4	2	= \$A4 + B\$1	= \$A4 + C\$1	= \$A4 + D\$1	= \$A4 + E\$1	= \$A4 + F\$1
5	3	= \$A5 + B\$1	= \$A5 + C\$1	= \$A5 + D\$1	= \$A5 + E\$1	= \$A5 + F\$1
6	4	= \$A6 + B\$1	= \$A6 + C\$1	= \$A6 + D\$1	= \$A6 + E\$1	= \$A6 + F\$1

Here's the result:

	A	B	C	D	E	F
1	+	0	1	2	3	4
2	0	0	1	2	3	4
3	1	1	2	3	4	5
4	2	2	3	4	5	6
5	3	3	4	5	6	7
6	4	4	5	6	7	8

## D. Graphs (Charts)

If you have a table of data, it's easy to graph it in Excel. Unfortunately, Excel is designed for business, not math teaching, so it sometimes does things that are mathematically annoying. For example, it will fit the graph into a certain shape of rectangle, which makes vertical and horizontal units of different sizes, and distorts the shape of the graph. Still, it's so easy that it's usually worth doing.

### Graphs from column (or row) tables in Excel.

Tables of values for functions are usually shown in a column table, with input values (x) on the left, and output values (y) on the right. Two function tables are shown below; Function 1's values are in a random order in the first table, and the same values are sorted in the second table.

Suppose we want to find if the points lie on a line.

Function 1, unsorted

x	y
0.24	0.0576
1	1
4.5	20.25
2.3	5.29
6	36
-3	9

Function 1, sorted with x values increasing

x	y
-3	9
0.24	0.0576
1	1
2.3	5.29
4.5	20.25
6	36

Function 2

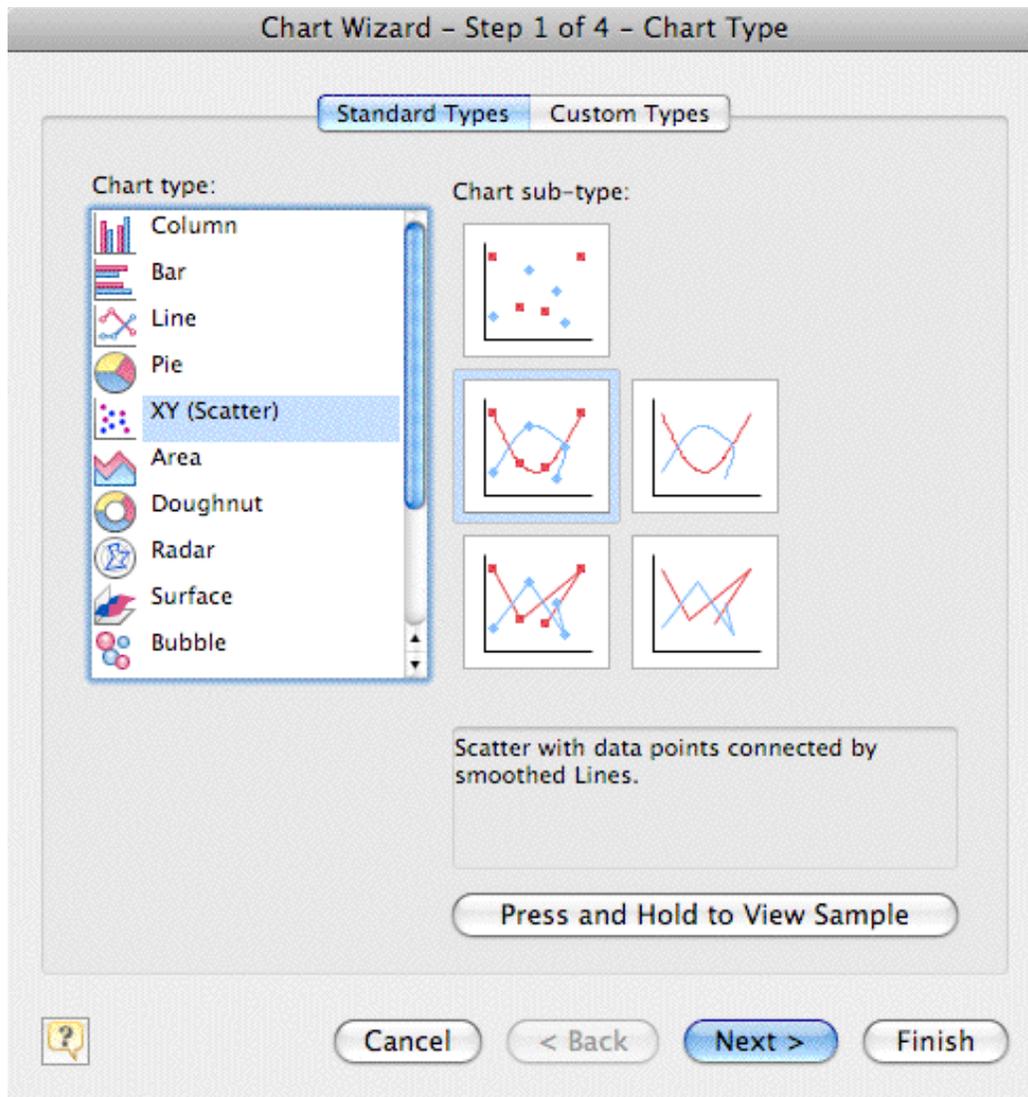
x	y
-3	-10
0.24	-0.28
1	2
2.3	5.9
4.5	12.5
6	17

To plot# these points,

1. Select all the cells of the table that contain data. If you select the headings, Excel will figure out that these are labels.



2. Click on the chart button on the top menu bar. You will get a series of choices.
3. Select chart type XY (Scatter). This will interpret the first column as x values and the second column as y values. Do NOT choose Line graph. This interprets all the values as y values, and uses the row number as the x value. Click whichever option you want: connect the points with line segments, with curves, or not at all. Warning: the points are connected in the order in which they occur in the table; see Figure 3 below. You can get a preview of the graph by clicking the button.



4. Further screens of choices allow you to change which cells will be graphed, to enter titles and labels for the graph, and whether you want the graph on a separate page.
5. For further formatting, control-click (or right-click) on the part of the graph you want to change: the graph points or lines, the background, or the back of the background. Then you can change colors, shapes of point markers, grid lines, labels for axes, titles, and even the range of data to be plotted and type of graph.

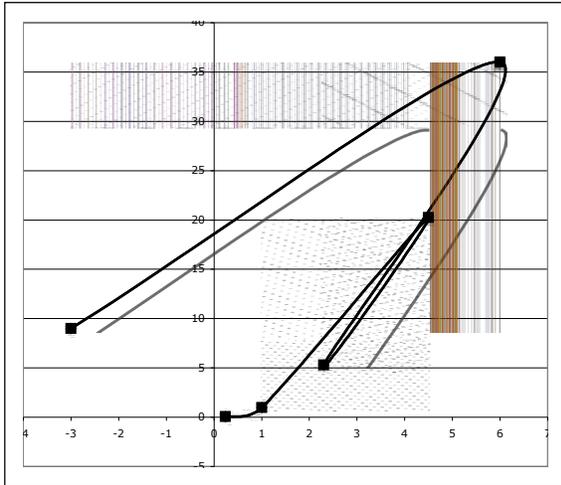


Figure 3. Function 1: unsorted data, connected with curves, makes a mess.

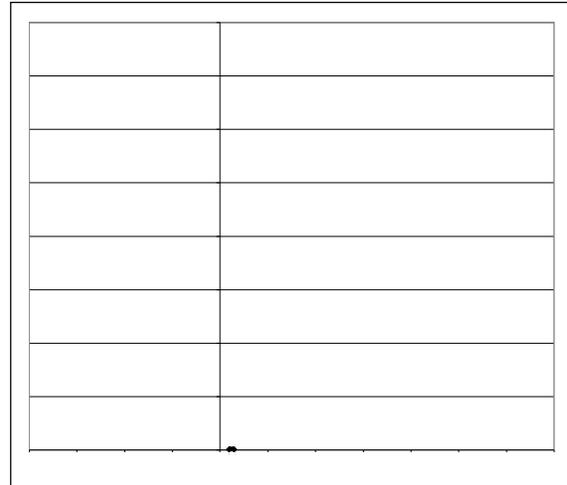


Figure 4. Function 1, just the points. Clearly not on a line.

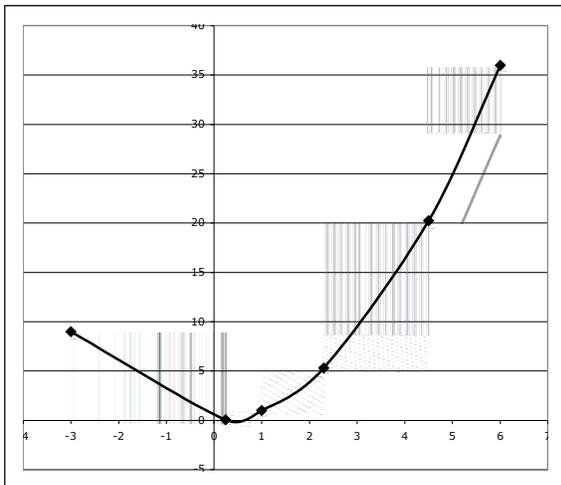


Figure 5. Function 1, sorted data. Might be a parabola?

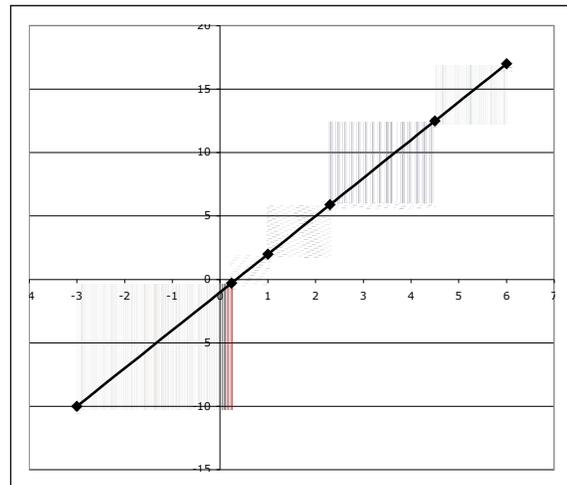


Figure 6. Function 2. Looks like a line!

### Graphs from grid tables in Excel.

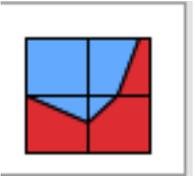
Grid tables, like the addition and multiplication tables, can also be graphed, though their graphs are 3-dimensional.

1. Set up your table so that headings are at the top and down the left. Alternatively, labels can be at the bottom, but don't select them.
2. Select all the cells of the table that contain data.

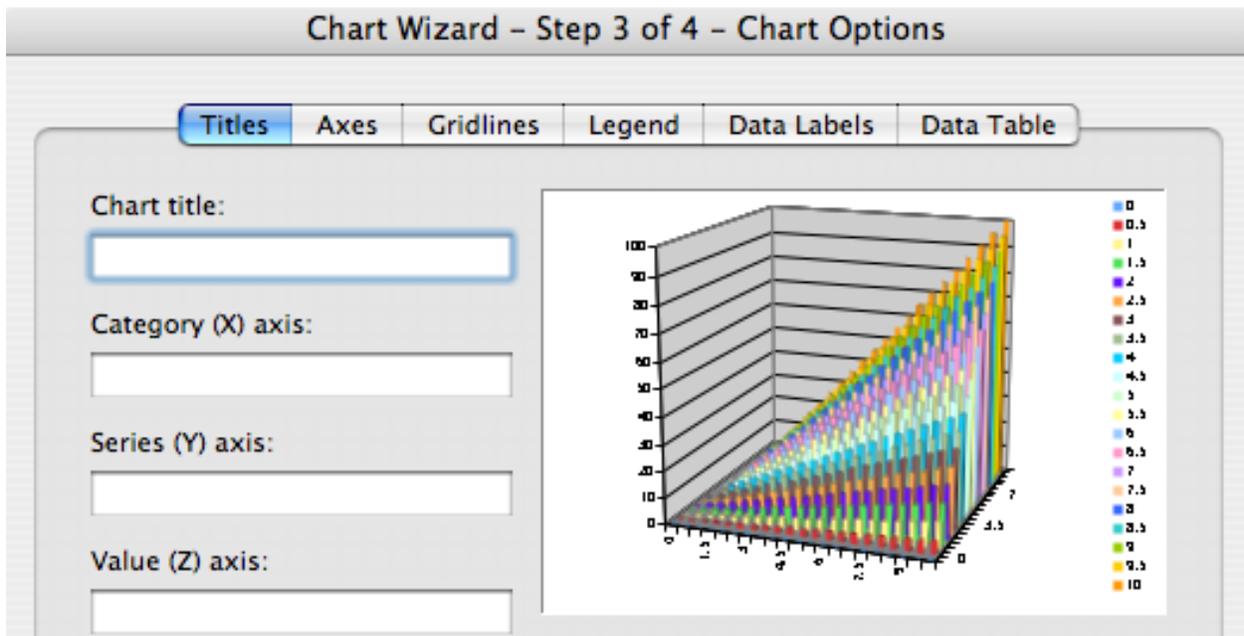
3. Click on the chart button on the top menu bar.



4. Choose one of the following types and subtypes:

Type	Subtype	Name
 Column		3d bar graph/chart. Each cell becomes a vertical bar, rising from the cell of the grid table.
 Surface		3d surface graph/chart. This gives a smoothed out version of the bar graph.
 Surface		Contour graph/chart. This gives a "topographic" map of the surface map, with different heights color coded. It's a 2-dimensional picture from above of the 3-dimensional surface.

5. You don't need to deal with the other steps right now; just say OK.



- To get a different view of the graph, click on one of the corners and drag the mouse. If you hold down the control key while doing this, you can see the outline of the graph while it's moving.
- Sometimes the graph will need tweaking. Select the graph, then use the Chart menu, or control-click on part of the graph. This works pretty much the same as in the other kinds of graphs.

With bar graphs, you can change the spaces between the bars (for mathematical purposes, you usually want no space.) Control-click on one of the bars, then Format Data Series -> Options. Then change Gap depth and Gap width to 0.

## E. Formatting

Formatting includes specifying the type of input in a cell, as well as appearances such as color and font and size of cells.

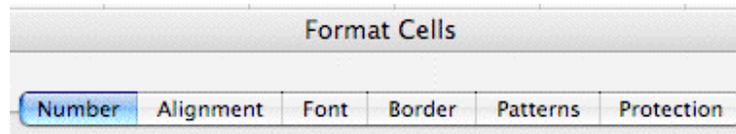


Figure 7. The Format -> Cells menu

### Number and other content formats.

To change the way numbers are displayed, either in a cell, a column, or in the entire worksheets, select the cells, then click the Number tab in the Format Cells window. Many of these commonly used commands can also be done by clicking an icon on a toolbar at the top of the screen.

You can display more or fewer decimal places (to show a rounded value.) Excel computes to many decimal places internally, but you don't necessarily want to see that information.

**Warning!** Excel computes to 13 decimal places internally, and no more. This is more than enough for practical work, but if you are looking for mathematical patterns, be suspicious if the digits are all 0 past a certain point.

Use the Currency choice to put in a dollar sign.

Note that Excel will also display and do computations with fractions, though it will handle only up to 3-digit denominators. You can also convert decimals (approximately) to binary fractions by choosing Fractions in sixteenths.

If you don't specify anything, Excel will choose General, in which anything containing a letter is treated as text, anything containing only digits (and a few other symbols such as period, comma, \$, and %) as numbers, and anything with a slash is treated as a date. This is a problem if you want an ID number that starts with a 0, such as 00125398. Excel will treat this as a number and change it to 125398. If you want a fraction such as 2/3, Excel will treat it as Feb. 3 unless you change the format to Fraction.

If you choose Text, you can make the text run vertically, have it shrink to fit the cell, or wrap it to form multiple lines when it is wider than the cell.

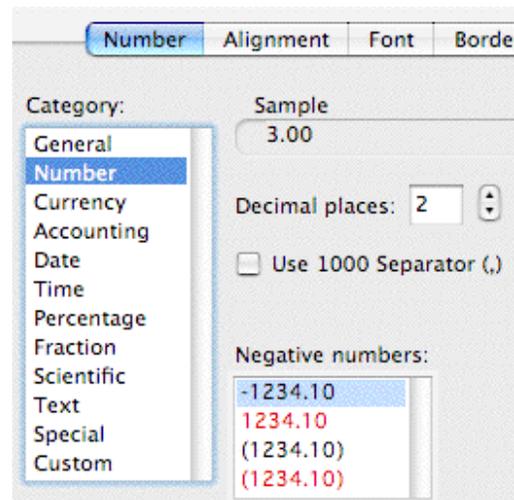


Figure 8. The Format -> Cells -> Number choices

## Fitting things in.

The sizes of cells, rows, and columns can be adjusted:

- manually, by dragging the dividers in the top label row (A, B, C, ...) or the left label column (1, 2, 3, ...). You can do several at once by selecting a range of rows or columns. Or you can do the entire sheet by clicking in the upper left corner (on the symbol <>).
- manually, by specifying exactly how big. Format -> Column -> Width, then type in a measurement such as 20 pt, .5 in, or 2 cm.
- automatically, to fit the contents: Format -> Column -> Autofit Selection

For text, choose Format -> Cells -> Text -> Alignment, then choose Wrap text if you don't want to change the column width. However, then the rows will be taller where there is a lot of text.

## Fonts, sizes, colors.

Don't spend a lot of time on cosmetic formatting right now; it's more important to learn the mathematical aspects of the program.

**Fonts** can be changed, as well as the size and colors of letters.

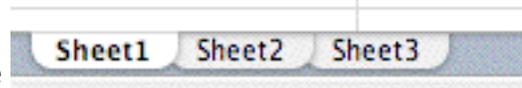
**Colors.** To color the background of a cell, click in the cell, then use the paint bucket on the formatting toolbar. Hold down the small triangle to the right of the bucket to choose a different color. If you don't see this toolbar, View -> Toolbars -> Formatting toolbar.



**Selecting many cells.** To avoid repeating the same formatting moves, you can select a range of cells in a strip or rectangle by dragging. You can also do this by holding down the shift key while clicking; this selects all the cells in between. If you want to select non-adjacent cells, hold down the option (Windows) or Apple (Mac) key instead of shift.

## Worksheets.

A spreadsheet can have many sheets. This is a good way to keep different, related tables together, rather than putting them all over one page, or making separate files. One use is to put separate problems of the same homework assignment in the same file. Another is to include the same tables with different parameters, such as scenarios for mortgages with different interest rates.



In Excel, files start with several sheets. You can switch between them with the tabs at the bottom of the page.

Change the name to something more descriptive than “Sheet 1” by double clicking on the tab.

If three sheets isn’t enough, add another using Insert -> Worksheet (screen snap below.)

If you would like to copy a sheet with minor changes to reuse your work, use instead Edit -> Move or copy sheet. A window comes up that allows you to change the order of the sheets. To copy an existing sheet, select the sheet on the list and check the Create a copy box. See screen snaps below.

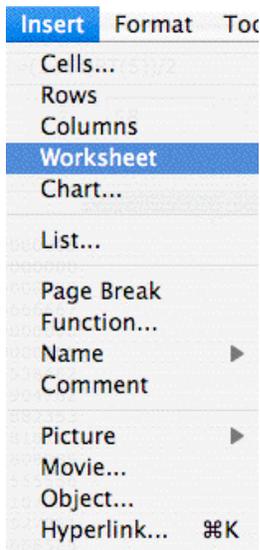


Figure 9. Insert sheet

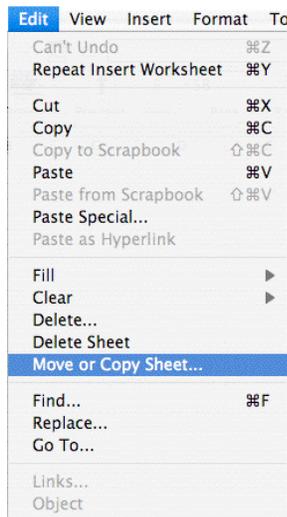


Figure 10. Move or copy sheet

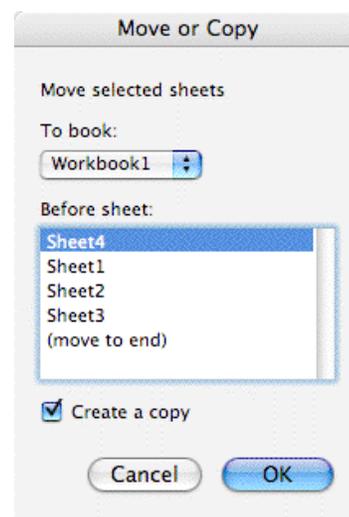


Figure 11. Choices for move or copy sheet

## F. Common mistakes in using spreadsheets

- Typing rather than using formulas. If your data has a pattern or relation to previously entered data, use a formula.
- Assuming the program can read your mind. While it may seem obvious to you that there is a pattern, and what the pattern is, the program may not have the same idea. Patterns can be subtle, and the program will only fill certain basic types of patterns. If in doubt, use a formula and fill the formula.

- Not checking that you gave the right instructions to do what you want. It's tempting to just move on after you have entered and filled a formula, but not checking can cause fatal errors. Mentally or manually compute what the first few results should be, as a spot check.
- You filled, changed the first cell, but forgot to change the filled cells. Suppose you entered a formula and filled it down. Then you decide to change the formula, and change it in the top cell. If you check, the formulas below it are still copied from the old formula. You have to fill the new formula down.
- Using relative cell references instead of absolute cell references. Suppose, in the cheese table, that the price of cheese sometimes changes, and you don't want to have to re-edit and fill the formula, as in the previous paragraph. You put the price of cheese in, say, cell A4, and use A4 instead of \$4.95 in the cost formula. (Try it.) However, when you fill down, A4 becomes A5, then A6, and so on. There is a fix for this problem: type \$A\$4 in the formula. This means to never change the row A or the column 4 when copying the formula. If you put the \$ sign in front of only one part of the address, such as A\$4, then the A will change when copied, but the 4 will not.
- Failure to RTM (Read The Manual.) All programs described have extensive help under the Help manual. For more obscure questions that aren't answered there, try a web search; there are many places that offer "how to's".

## G. Free software

OpenOffice.org is a multiplatform and multilingual office suite and an open-source project. Compatible with all other major office suites, the product is free to download, use, and distribute. <http://www.openoffice.org/>

Google Documents is found at <http://docs.google.com>. They continue to add features, but at this writing, Google spreadsheet will do less than the other two programs described.

GeoGebra can be downloaded from <http://www.geogebra.org>

## H. Problems and Exercises

Except where stated, use a spreadsheet. Your homework should be a single file with each problem on a separate sheet. Double click on the tab at the bottom to label the sheet with the problem number. Turn in the homework on Moodle in the Activities/Assignments section by uploading your Excel file. If you are using OpenOffice, save as Excel. The file name should come out ending with ".xls".

### Exercises

1. Use a spreadsheet to fill in this table for cheese.com up to 15 pounds of cheese, where aged English cheddar cheese costs \$7.65 per pound, and there is a flat \$4.50 shipping fee per order. Enter a formula into the shaded cell, then fill down to complete the table. You may want to compute the total in two steps, with an extra column.

pounds of cheddar	total cost of order
1	
2	
3	
4	

Use as little typing, and as much filling, as possible. When finished, color in with the paint bucket the cells that you had to type in something explicitly. Leave white the cells you filled in without typing.

- Similar instructions to exercise 1, except compute the total cost for restaurant dinners with a 7.5% tax, and a 15% tip on the total after tax. Make up 4 random dinner costs (before tax and tip).
- Fill in x values at least from -2 to +2 in steps of 0.1 in the first column. Then, for each of the next four columns, enter formulas for the functions shown below. (They won't look nicely formatted in your spreadsheet). As x goes from -2 to 2, what happens to the output values for each function? Do those values go up, too? or down? or up, then down, or something else? Explain.

x	$f_1(x) = 4(1-x)$	$f_2(x) = 3x^2 - x - 3$	$f_3(x) = \frac{5x-2}{3x-1}$	$f_4(x) = \sqrt{1-x^2}$
-2				
-1.9				

### Problems

- For the function  $f_3(x) = \frac{5x-2}{3x-1}$  in problem 3, something weird happens between 0.3 and 0.4. "Zoom in" on this region by using the same formula, but go up by hundredths (0.01) from 0.3 to 0.4. Then find more precisely where the weirdness happens, then go up by thousandths. Then zoom in some more, and go up by ten thousandths. What is happening with this function in this region, and how could you tell from its formula what is happening?
- Make a list of the decimals for unit fractions 1/1, 1/2, 1/3, 1/4, and so on. (Format the column of the fractions as fractions, and copy the same numbers in the next column, formatted as decimals.) Look over the numbers and make some conjectures about the patterns you see.
- Pascal's triangle is a pattern of numbers. The first few rows are shown below.

			1		
		1		1	
	1		2		1
	1	3		3	1
1	4	6	4	1	

Instead of being a sequence of numbers, it's a sequence of sequences. The rule for generating the rows is:

Each row starts and ends with 1.

Each middle entry is the sum of the two entries above it.

- a. Construct a spreadsheet that calculates at least 20 rows of Pascal's triangle.

```

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

```

It's easier to do if you have all the rows start in column 1 of the spreadsheet. Hint: find a formula for the 2 in the second row. Copy it manually to the first position in each column that should have a number (right 1, down 1). Then fill down the columns. Hint: if you add the contents of a blank cell to something, it's the same as adding 0.

- b. Insert a column before the first. Use it to contain the sum of that row of Pascal's triangle. Describe the pattern in the first column in some other way, besides "the sum of the nth row of Pascal's Triangle."
7. The *Fibonacci numbers*. The 0<sup>th</sup> Fibonacci number is 1, the 1<sup>st</sup> Fibonacci number is also 1, and each successive Fibonacci number is the sum of the two previous ones. So the first 5 of them are 1, 1, 2, 3, 5.
- a. Enter a formula for generating the Fibonacci numbers, then fill it down. You should only have to type the two 1s and one formula. (Your data will fit better on the screen if you list the Fibonacci numbers vertically.) Find at least 40 numbers in the sequence.
- b. Find the ratio of each Fibonacci number to the previous one, again using a formula and filling.
- c. Use guess and check or algebra to find two solutions of the equation  $x^2 - x - 1 = 0$ .
- d. The *golden ratio* is the number  $\frac{1+\sqrt{5}}{2}$ . Find a decimal for this number.
- e. How are the parts of this problem related?

## Projects

8. A *geometric sequence* is a list of numbers generated by repeatedly multiplying by a fixed number. Here is an example: 1, 2, 4, 8, 16, ... The sequence starts with 1, and to get each successive term, multiply by 2. The numbers are sometimes called *terms*. Sometimes the number that starts the list is called the first term, but you could also start counting with 0 (so 1 is the 0<sup>th</sup> term), or with any other whole number.

Use a spreadsheet to do many examples, starting with different first numbers, and using different multipliers, to find a one-step method to find the sum of the first K numbers in any geometric sequence. For example, the sums of the first K terms of the sequence 1, 2, 4, 8, ... are shown in the table below. Your job is to find a formula or method so that if you are given the initial term, the multiplier, and the number of terms to add, you can get the total without actually adding them all.

K	Kth term	sum of first K terms
0	1	1
1	2	3
2	4	7
3	8	15

Here are some smaller problems to work on. You don't have to answer the big problem in full generality, but do try to find a general pattern/rule/method.

- a. Initial term 1, multiplier 2
- b. Various other initial terms, multiplier 2. If you know how to find the sum with initial term 1, how does changing the initial term change the sum?
- c. Initial term 1, multiplier 10
- d. Various other initial terms, multiplier 10. If you know how to find the sum with initial term 1, how does changing the initial term change the sum?
- e. Multipliers  $1/2$ ,  $1/10$ .