Parabolas and Coordinates

Try this (1).

a) Consider the parabolas in Figures 1, 2, 3. For each figure, construct a coordinate system so that the curve as drawn has the equation $y = x^2$. Verify empirically by measurement that this equation holds for all points along the curve.

b) How would you vary that coordinate system so that the curve in Figure 3 has the equation $y = \frac{x^2}{3}$. Verify your new system empirically.

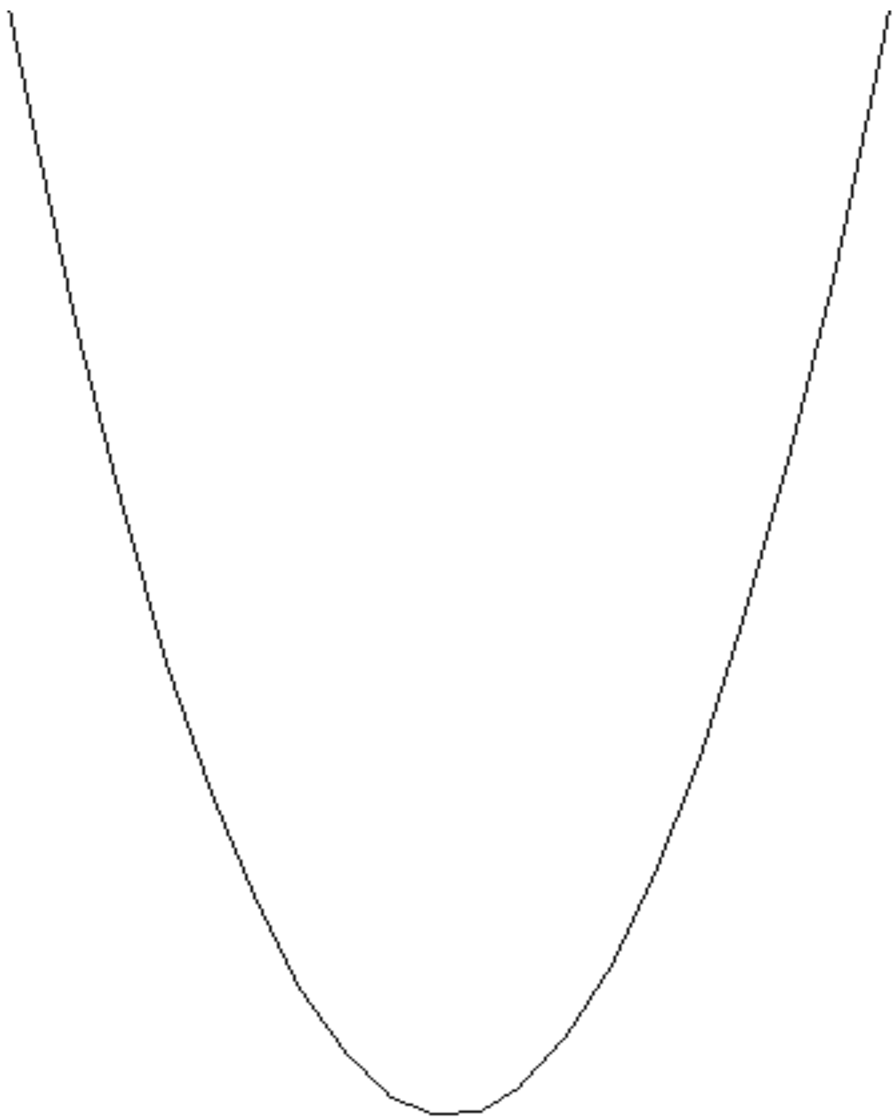

Figure 1

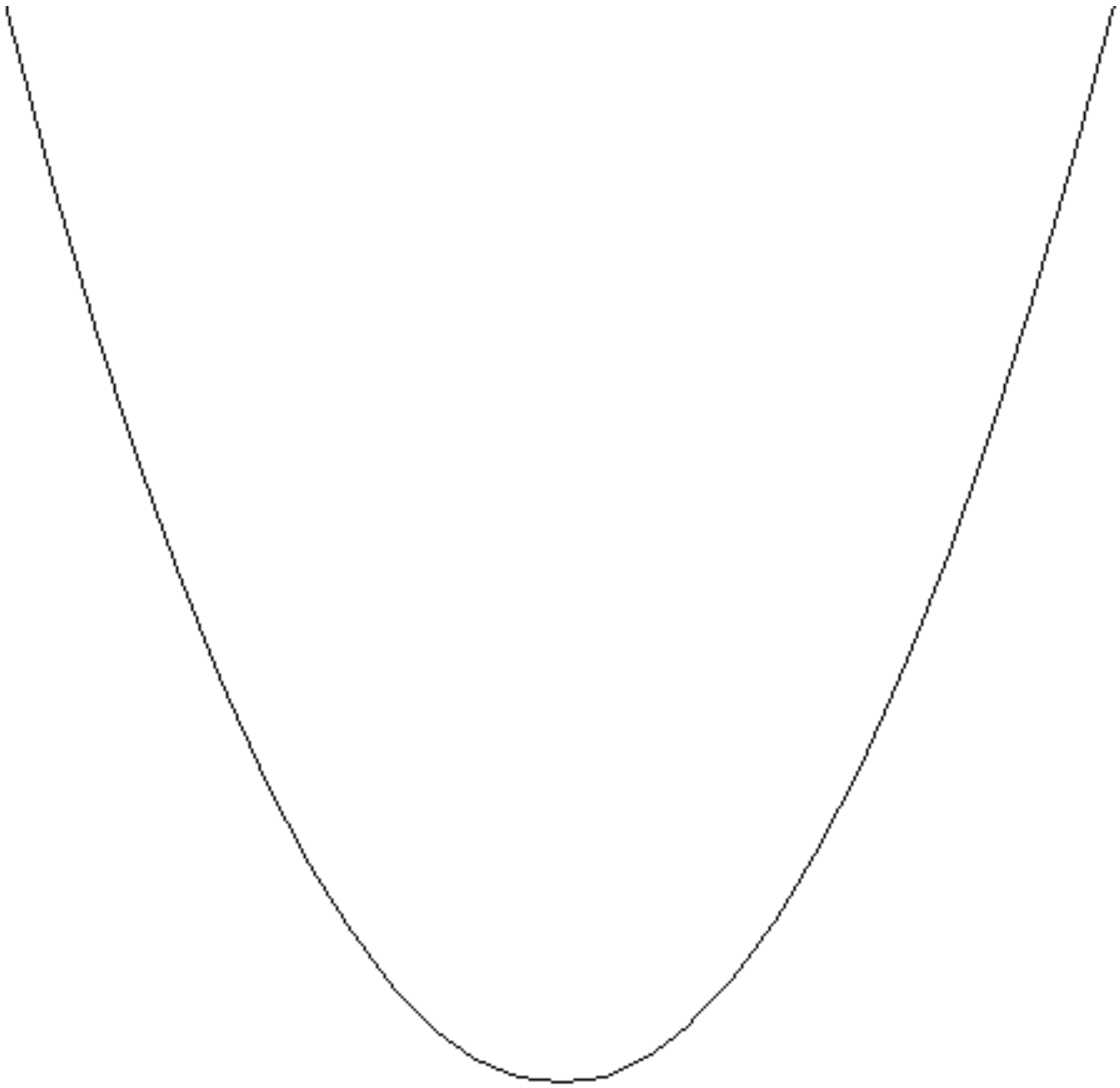

Figure 2

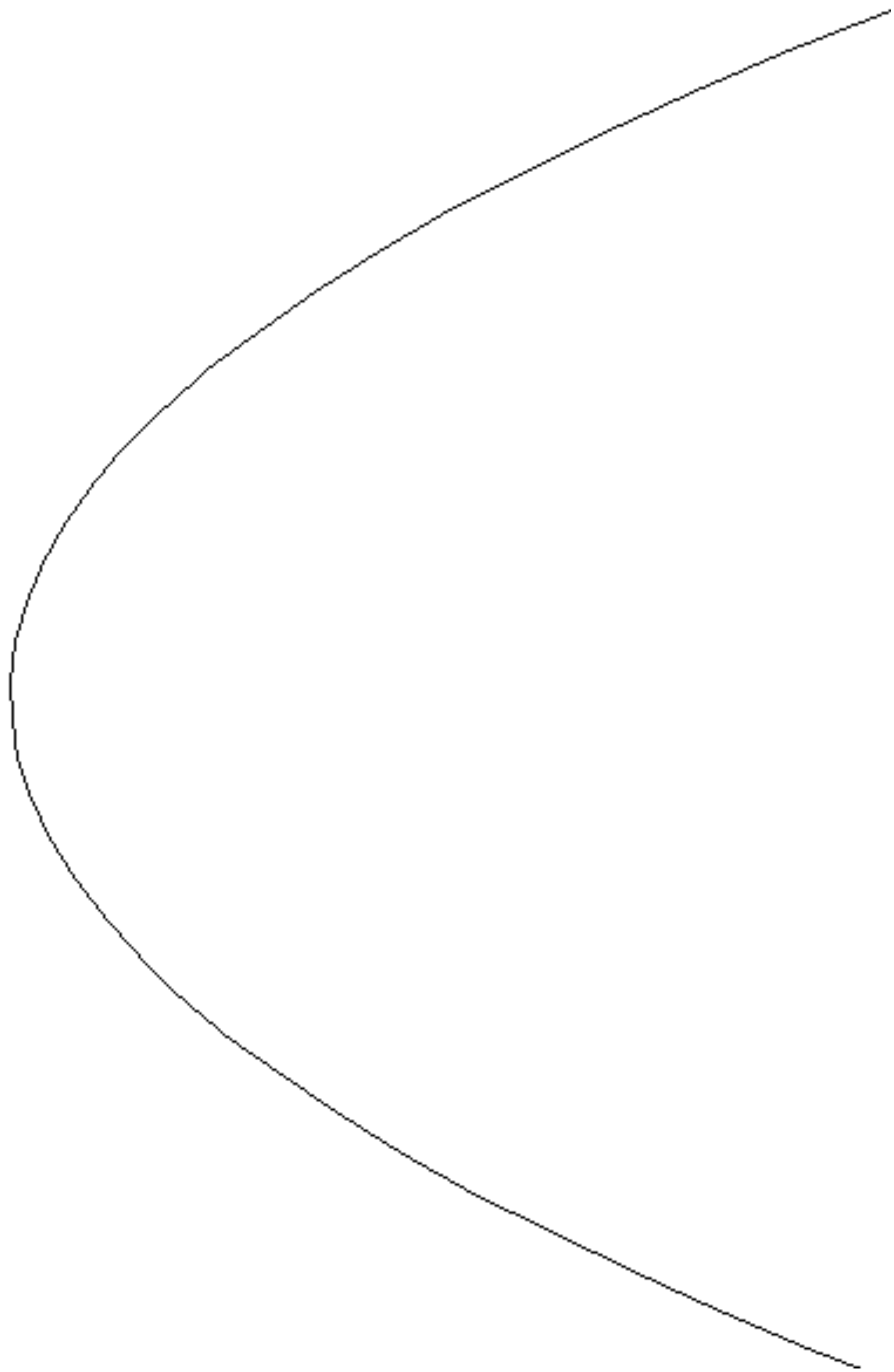

Figure 3

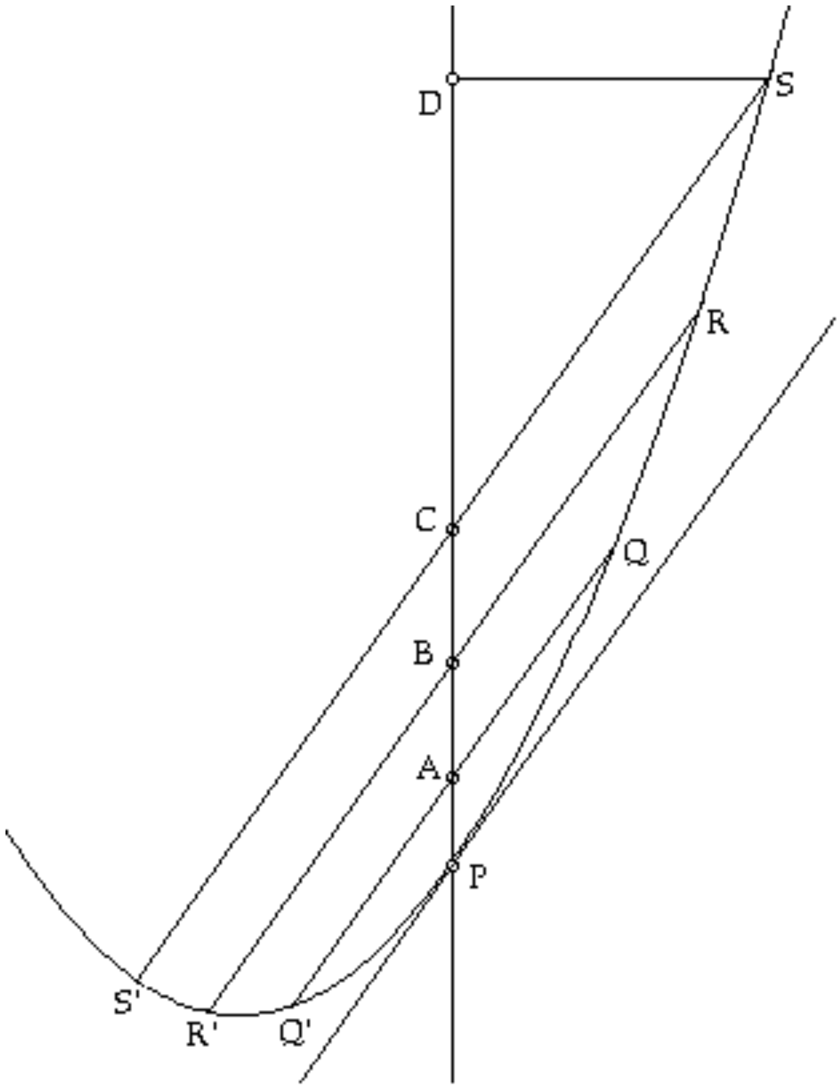

Figure 4

Figure 4 displays several properties of parabolas that were well known to Apollonius. Take any chord SS' and let C be its midpoint. Let Chords RR' and QQ' be any chords parallel to SS', with midpoints B, and A. Then C, B, and A will be collinear and the line through these midpoints will be parallel to the axis of symmetry of the parabola. Let P be the point where the line through CBA meets the curve. Then the tangent to the curve at P will be parallel to the chords SS', RR', and QQ'. Furthermore if we consider the lengths PA, PB, PC, and the corresponding lengths AQ, BR, CS, as a set of skewed coordinates for the curve then the vertical distance from P will be proportional to the square of the distance out to the curve in the direction of the chords (using this parallel family of chords).

c) Test the statements above empirically using the curves given in Figures 1, 2, 3. Can you give some reasons as to why these properties are characteristic of all parabolas?

d) If you wanted to physically trace, draw, or create a parabola, how might you go about doing this? What properties of the parabola are inherent in your method of generation?