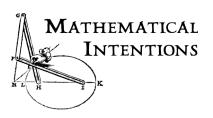
Napier Rods



John Napier (1550-1617) computed and published the first table of logarithms. He also invented a set of carved rods to make multiplication easier. They were sometimes made of bone or ivory, and so were sometimes called "Napier's Bones."

Each rod is a column of the multiplication table.

Example 1. Multiplying a multidigit number by a 1-digit number: 8 × 769.

- a. Get out the rods for the 7, 6, and 9 columns of the multiplication table, in order. Use the strip without diagonals on the left.
- b. Find the 8th row, using the strip on the left.
- c. Read off the answer, adding as you go. Place values are separated by the diagonals.
 The 8th row 56, 48, 72. The 56 is really 5600, since it was 8x700. Similarly, the 48 is really 480. The 72 is just 72.

In the total, there are 2 ones, 7+8 tens, 6+4 hundreds, and 5 thousands, before regrouping. This amounts to 6 thousands, 1 hundred, 5 tens, and 2 ones, so the product is 6152.

Try this (1).

- a) Figure out how to multiply two multidigit numbers, such as 657x789. You will probably want to do the final addition in writing.
- b) How do you decide where to put the decimal point when multiplying decimals? For example, 0.000328×0.017 or 926×0.000028
- c) Look up the Lattice, or Gelosia, method of multiplying, which was widely used until the printing press. How is it related to Napier rods?

9	6/3	54	8
8	56	4 8	7/2
7	4⁄9	4⁄2	⁶ /3
6	4⁄2	3⁄6	54
5	3 5	3⁄0	4 5
4	2/8	2⁄4	36
3	2/1	1/8	2⁄7
2	14	1/2	1 8
1	0⁄7	0⁄6	0⁄9
0	0⁄0	0⁄0	0⁄0
	7	6	9

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Napier rods, from Mathematicial Intentions: http://www.quadrivium.info