Drawing Ellipses and Finding Tangents

MATHEMATICAL INTENTIONS

Figures 1-6 below depict a variety of devices <u>all of which draw ellipses</u>. Figure 1 is the more familiar loop of string and two tacks which is generally discussed in high school. All of these figures are taken from a popular seventeenth century text by Franz van Schooten originally published in 1657 entitled: *Organica Coniccarum Sectionum in Plano Descriptione* (Organs for the Description of Conic Sections in the Plane). Van Schooten was a Dutch friend of Descartes who published an edition of Descartes' *Geometry* with extensive commentaries. Van Schooten's edition of Descartes was much more popular than the original and was one of the first mathematical works read by Newton along with this work of conic sections.

Try this (1).

a) Give some reasons why the devices in Figures 1 and 6 would trace the same curves. (Hint: What can you say about triangles HIE, and EFG?)
b) Give some reasons why the devices in Figures 6 and 7 would trace the same curves. (Hint: Consider the perpendicular bisector of GI in Figure 6.)
c) Describe the geometrical concept which underlies the device in Figure 7. (Hint: Consider the circle described by point G around center H.)
d) Consider the tangent property of parabolas that you investigated in Project #3, part c. Is there any property like that which works on ellipses? Draw some ellipses and test your ideas empirically.
e) What do Figures 6 and 7 say about the tangents to the ellipse? Describe your observations in geometrical language.
f) Try to give some reasons for any of your statements in parts d and e.
g) What relations between geometry and algebra are you assuming in you discussions?

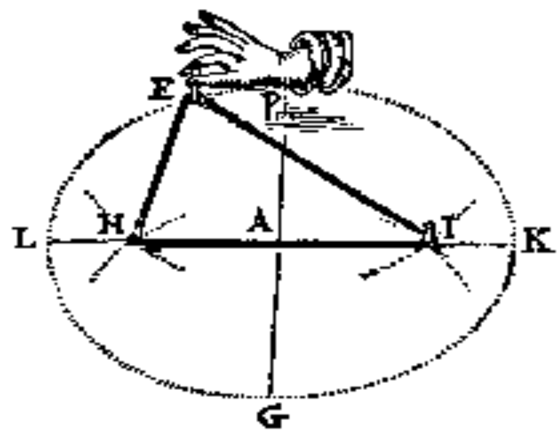


Figure 1

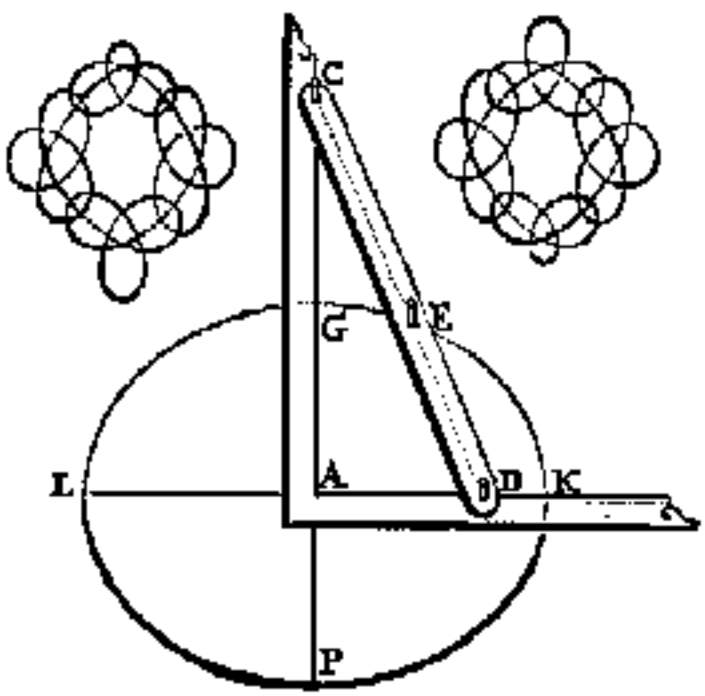


Figure 2

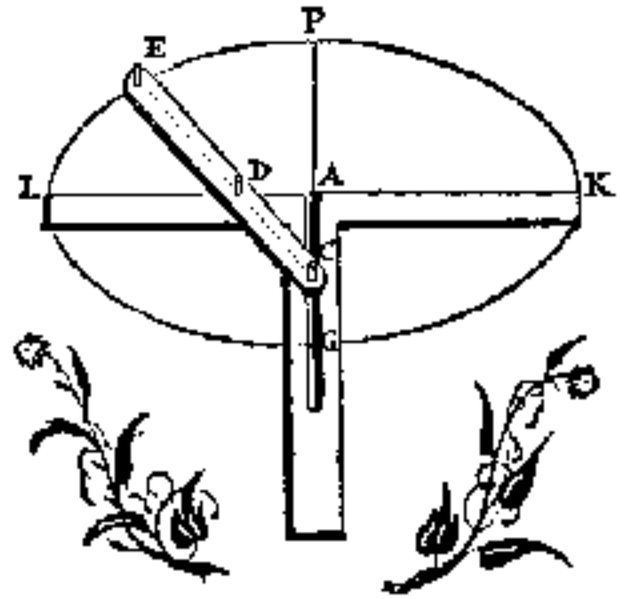


Figure 3

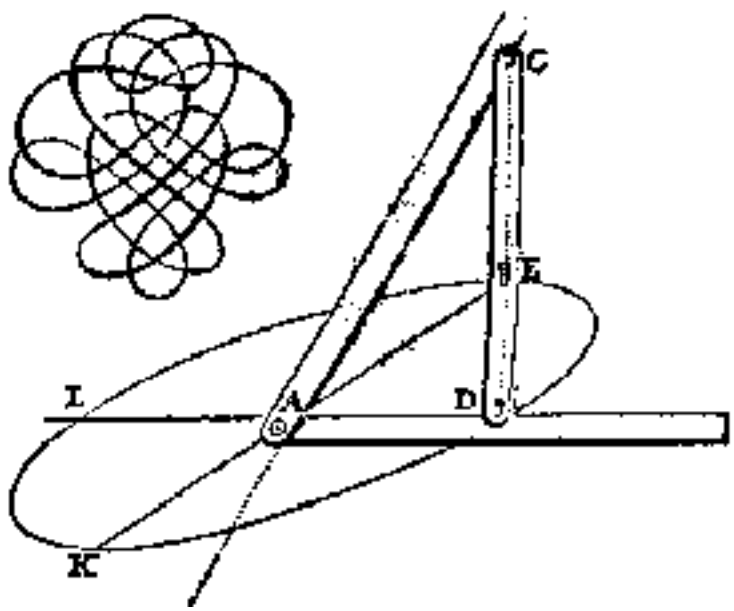
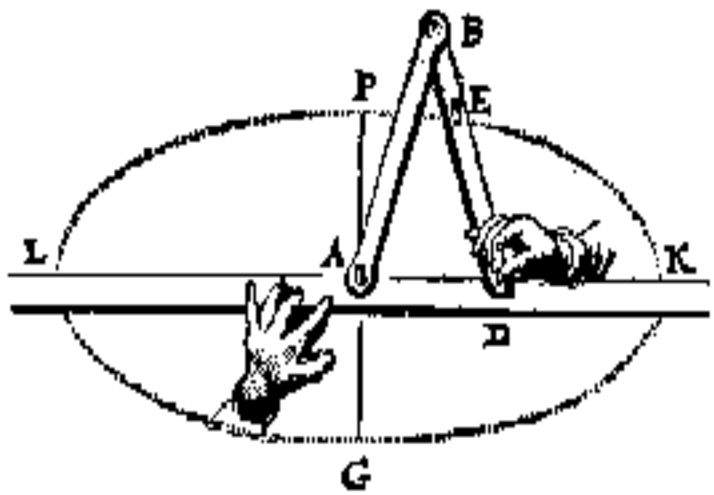
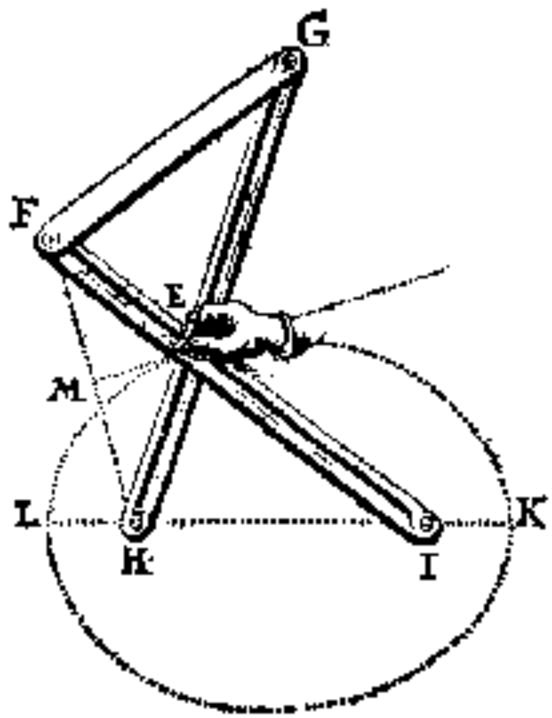


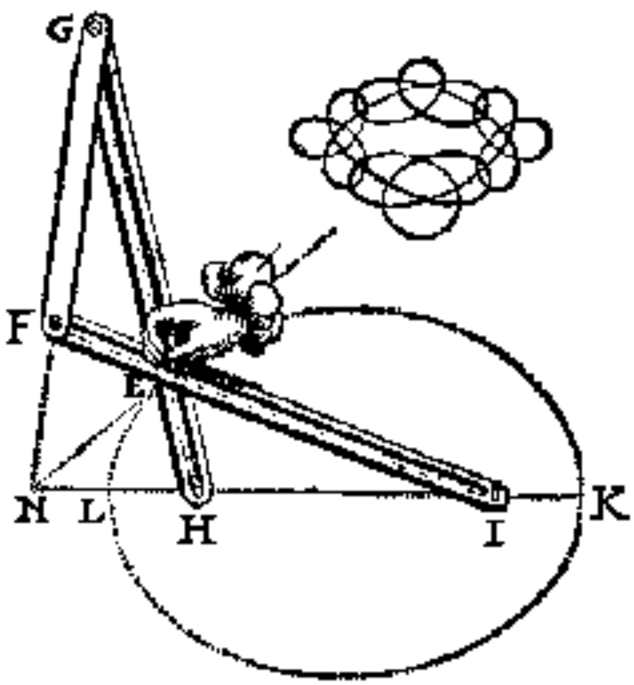
Figure 4



AB = BD

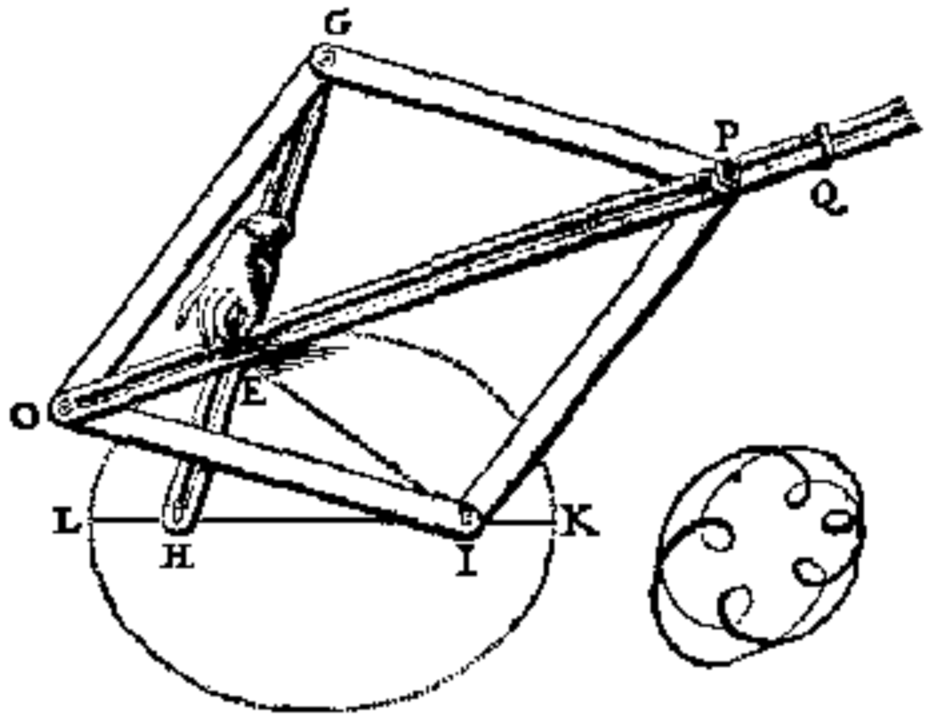
Figure 5





HG = FI & FG = HI

Figure 6



IO = OG = GP = PI

Figure 7